Name: ____________________________ Date: ___________

AP Calculus BC February Break 2015 Assignment

Parametric Equations

<u>Part 1 – What are Parametric Equations?</u>

Parametric equations are equations that model the position (in terms of x and y) of an object with respected to a 3^{rd} variable, called a parameter (typically this is "t").

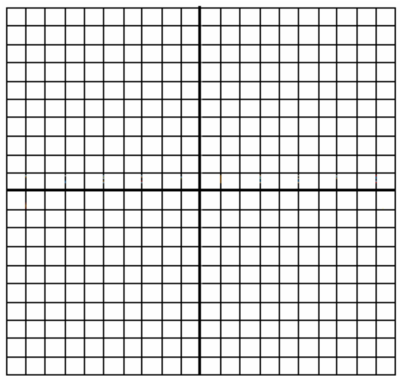
Example:

$$P(t) = \begin{cases} x(t) = t^2 - 2t \\ y(t) = t + 1 \end{cases} \quad for\ t \in (-2, 4)$$

In this example, P(t) will have two components, an x-coordinate and a y-coordinate, both found by evaluating the respective function at values of t in the domain.

To understand this better, complete the table of values below and graph your results:

t	$x(t) = t^2 - 2t$	$y(t) = t + 1$
-2		
-1		
0		
1		
2		
3		
4		



<u>Questions:</u>

1.) Describe the shape of the curve that you graphed.

2.) What is the initial position of this object (within the given domain of t)?

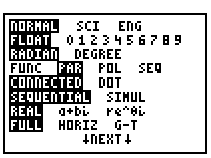
3.) What is the ending position of this object?

4.) Does this object change direction? If so, where?

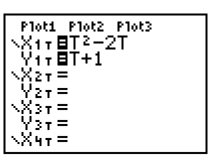
5.) When (over what interval) is this object moving to the right? Left?

6.) When is this object moving up? Down?

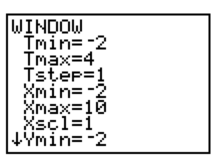
Now, we're going to analyze this same function on our graphing calculators:



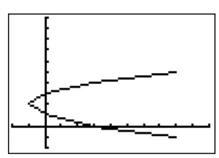
Change mode to PAR



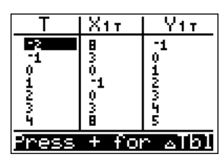
Enter the equations



Set window for t, x & y



Graph it!



Look at the table.

In summary, parametric equations model the motion of particles in a 2 dimensional plane according to a parameter such as time. This is a powerful idea, since it allows us to address movement in multiple directions according to time.

Part 2 – What Calculus can you do with Parametrics?

Here's the calculus you need to be able to do with parametric curves on the BC Exam:

1. Find $\frac{dy}{dx}, \frac{dx}{dt}$ and $\frac{dy}{dt}$
2. Find equations for tangent lines
3. Find times and position of certain key events (changes in direction)
4. Find the lengths of curves defined parametrically
5. Find the speed of a particle defined parametrically
6. Find the acceleration vector of a particle

Let's examine each of these.

1.) Find $\frac{dy}{dx}, \frac{dx}{dt}$ and $\frac{dy}{dt}$

Using P(t) from the 1st page, find $\frac{dx}{dt}$ and $\frac{dy}{dt}$

$\frac{dx}{dt} =$

$\frac{dy}{dt} =$

The formula for $\frac{dy}{dx}$ is given by $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$$\frac{dy}{dx} =$$

2.) Once you know $\frac{dy}{dx}$, all you need are an x and a y coordinate to find the equation for a line tangent to the curve.

Find the equation for the line tangent to the curve when t = 2. Graph it on the axes on page 1.

3.) Find the time and position of changes in direction and extreme values.

What kind of tangent line would exist at a vertical change in direction, vertical or horizontal?

What kind of tangent line would exist at a horizontal change in direction, vertical or horizontal?

Horizontal tangent lines occur when $\frac{dy}{dx} = 0$, meaning $\frac{dy}{dt}$ = 0 and $\frac{dx}{dt} \neq 0$.

Vertical tangent lines occur when $\frac{dy}{dx}$ is undefined, meaning $\frac{dx}{dt}$ = 0 and $\frac{dy}{dt} \neq 0$

Just like before, extreme values occur at critical points and endpoints (of the interval for "t"). What are the vertical and/or horizontal extreme values for P(t)?

4.) We have previously learned a formula to find the length of an arc in a curve. This formula is:

$$\int_a^b \sqrt{1 + f'(x)^2}\, dx$$

A similar formula exists for parametric curves. We use the letter "s" to denote the length of the curve:

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Sometimes you will be able to evaluate this integral by hand, sometime it will require the use of a calculator.

Find the length of the curve P(t) from t = -1 to t = 3

5.) Finding the speed of the particle:

In parametric equations, speed is the derivative of the curve length with respect to time.

Using the Fundamental Theorem of Calculus, we know can find the formula for speed, $\frac{ds}{dt}$:

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

Find the speed of the particle at the following times:

@t = -2:	@t = 2
@t = 0:	@t = 3:
@t = 1:	@t = 4:

When is the particle speeding up? When is it slowing down?

6.) Finding the acceleration vector of a particle:

The acceleration vector is given by the formula $[\![x(t)'', y(t)'']\!]$, in other words, evaluate the 2nd derivative of x and y at some t value and write your answer as a coordinate pair.

Find the acceleration vector of P(t) when t = 2:

Part 3 – Sample AP Problems – Multiple Choice and Free Response

a.) Sample problems – Multiple Choice

1997 #2

2. If $x = e^{2t}$ and $y = \sin(2t)$, then $\frac{dy}{dx} =$

(A) $4e^{2t}\cos(2t)$ (B) $\frac{e^{2t}}{\cos(2t)}$ (C) $\frac{\sin(2t)}{2e^{2t}}$ (D) $\frac{\cos(2t)}{2e^{2t}}$ (E) $\frac{\cos(2t)}{e^{2t}}$

1997 #18

18. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only

(B) 1 only

(C) 0 and $\frac{2}{3}$ only

(D) 0, $\frac{2}{3}$, and 1

(E) No value

1998 #10

10. A particle moves on a plane curve so that at any time $t > 0$ its x-coordinate is $t^3 - t$ and its y-coordinate is $(2t - 1)^3$. The acceleration vector of the particle at $t = 1$ is

(A) (0, 1) (B) (2, 3) (C) (2, 6) (D) (6, 12) (E) (6, 24)